Near-Optimal Approximate Shortest Paths and Transshipment in Distributed and Streaming Models

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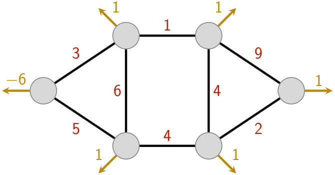
Transshipment & Single Source Shortest Path

Given **undirected** graph $G = (V, E)$
- edge **weights** $w \in \mathbb{N}^m$,
- node **demands** $b \in \mathbb{Z}^n$ s.t. $\mathbb{1}^T b = 0$



Undirected Transshipment

$$\min\{\|Wx\|_1 : Ax = b\} = \max\{b^T y : \|W^{-1}A^T y\|_\infty \leq 1\}.$$

Single Source Shortest Path is special case $b = \mathbb{1} - n\mathbb{1}_s$.

Lemma

The problem $\min\{\|W^{-1}A^T\pi\|_\infty : b^T\pi = 1\}$ is equivalent.

Approximately solve by

minimizing $\Phi_\beta(\pi) := \mathrm{lse}_\beta(W^{-1}A^T\pi)$, where $\mathrm{lse}_\beta(v)$ is log-sum-exp.

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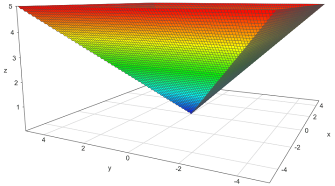

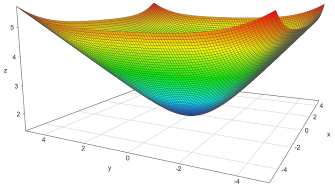

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Approach: Gradient Descent on $\Phi_\beta(\pi)$

Potential Function Change on Update

$$\Phi_\beta(\pi) - \Phi_\beta(\pi - h) \geq \nabla\Phi_\beta(\pi)^T h - \beta\|W^{-1}A^T h\|_\infty^2$$

- This suggests to compute h by solving

$$\max\{\nabla\Phi_\beta(\pi)^T h : \|W^{-1}A^T h\|_\infty \leq 1\}.$$

- **Another transshipment problem** with demand vector $\nabla\Phi_\beta(\pi)$.
- Using **oracle** for α**-approximation** h yields:

Theorem

Using oracle for α-approximate transshipment, one can compute solutions x, y s.t. $\|Wx\|_1 \leq (1+\varepsilon)b^T y$ with $\tilde{O}(\varepsilon^{-3}\alpha^2)$ oracle calls.

(Truth a bit more complicated: The feasible direction h must satisfy $b^T h = 0$.)

Results: Distributed/Streaming Models

α-oracle

Use the optimal solution on sparse α-**spanner**.

	Previous	**New**
Brdc Congest SSSP	$\varepsilon^{-o(1)}(n^{1/2+o(1)} + D^{1+o(1)})$ rounds	$\tilde{O}(\varepsilon^{-O(1)}(n^{1/2} + D))$ rounds
Brdc Congest Clique SSSP/STP	$\varepsilon^{-o(1)} n^{o(1)}$ rounds	$\tilde{O}(\varepsilon^{-O(1)})$ rounds
Multipass streaming SSSP/STP	$\varepsilon^{-o(1)} n^{o(1)}$ passes	$\tilde{O}(\varepsilon^{-O(1)})$ passes

- **Matches lower bounds** in terms of n up to log-factors.
- Provides hope towards **parallel solutions** of SSSP/STP?

Thank you for your attention!