

(Wireless) Scheduling, Graph Classes, and c -Colorable Subgraphs ¹



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Abstract

The NP-hard problem of finding maximum-weight induced c -colorable subgraphs, which is a generalization of finding maximum independent sets, naturally occurs when selecting c sets of pairwise non-conflicting jobs (modeled as graph vertices). We investigate the parameterized complexity of this problem with respect to the solution size on inductive k -independent graphs. Inductive k -independent graphs are a generalization of chordal graphs and have recently been advocated in the context of interference-avoiding wireless communication scheduling [Ásgeirsson et al, ICALP, 2017 [1] and Halldórsson and Tonoyan, STOC, 2015 [4]]. We show that the Maximum Independent Set problem is W[1]-hard even on 2-simplicial 3-minoes—a subclass of inductive 2-independent graphs. In contrast, we prove that the more general c -COLORABLE SUBGRAPH problem is fixed-parameter tractable on edge-wise unions of cluster and chordal graphs, which are 2-simplicial. Aside from this, we survey other graph classes between inductive 1-independent and inductive 2-independent graphs with applications in scheduling.

Introduction

Scheduling, routing and power control/assignment in wireless communication networks translates to c -COLORABLE SUBGRAPH in inductive k -independent graphs with constant k [1].

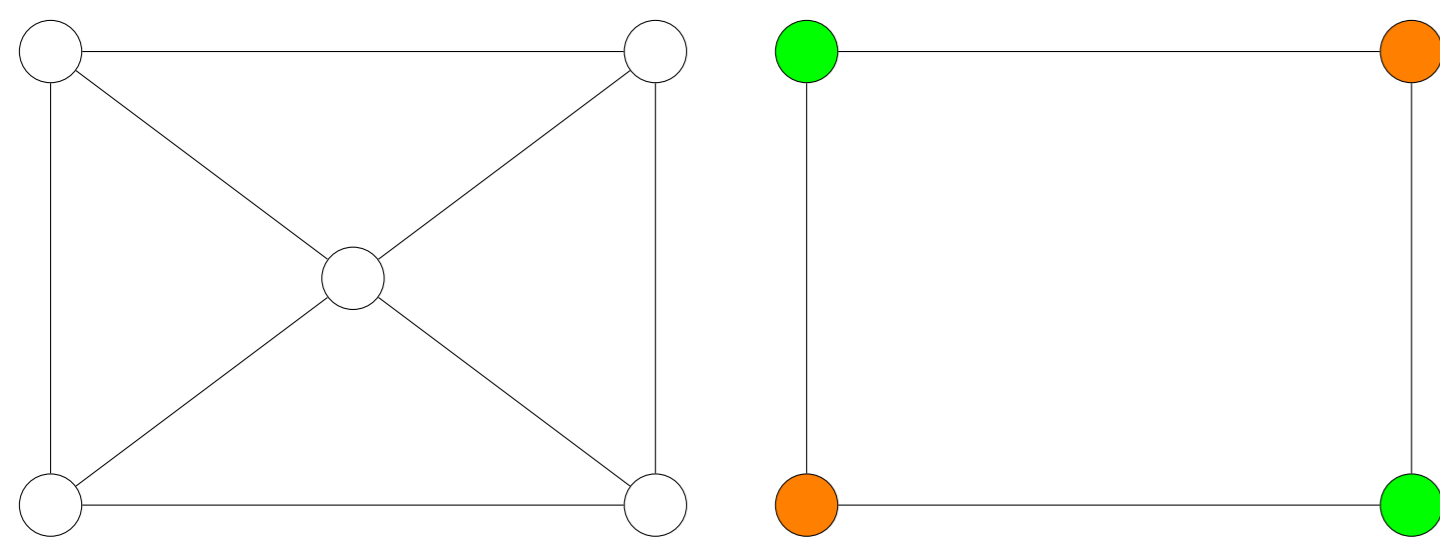
c -COLORABLE SUBGRAPH

Input: A graph $G = (V, E)$ (possibly with vertex weights).

Question: Find a set $S \subseteq V$ of maximum cardinality (weight) such that $G[S]$ is c -colorable.

1-COLORABLE SUBGRAPH = MAXIMUM INDEPENDENT SET

Example ($c = 2$):



Related Work

- MAXIMUM INDEPENDENT SET parameterized by solution size is fixed-parameter tractable on strip graphs [3] (= cluster \bowtie interval graphs).
- MAX-WEIGHT INDEPENDENT SET parameterized by solution size is W[1]-hard on unit 2-track interval graphs [5] (= interval \bowtie interval graphs).
- (Unweighted) c -COLORABLE SUBGRAPH parameterized by solution size is fixed-parameter tractable on inductive 1-independent (= chordal) graphs [6].

Overview Of Results

Summary of results (both new and previously known):

Legend for diagram:

green — c -COLORABLE SUBGRAPH in polynomial time.

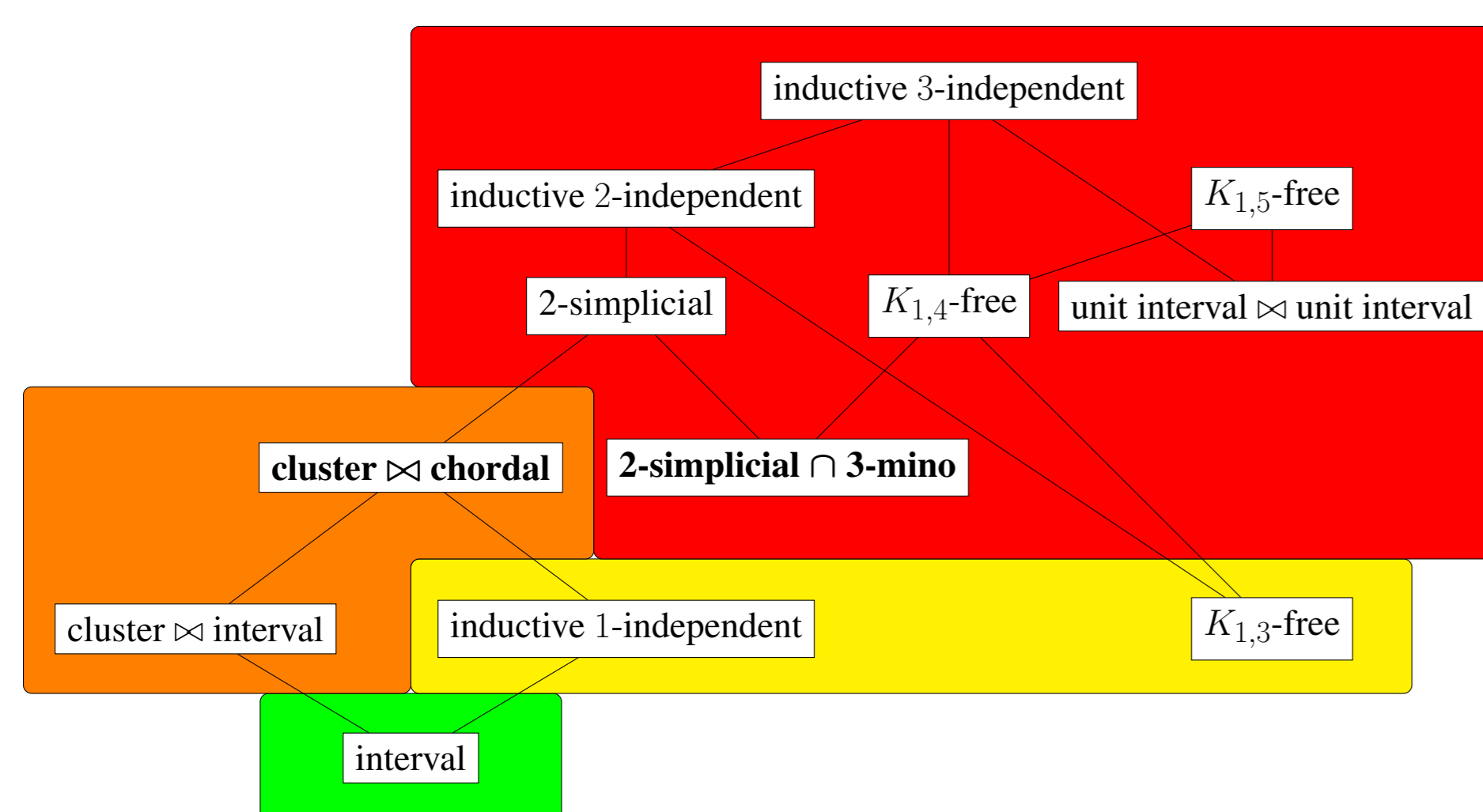
yellow — MAX-WEIGHT INDEPENDENT SET in polynomial-time, c -COLORABLE SUBGRAPH NP-hard but fixed-parameter tractable

orange — MAXIMUM INDEPENDENT SET NP-hard, c -COLORABLE SUBGRAPH fixed-parameter tractable

red — MAXIMUM INDEPENDENT SET NP-hard and W[1]-hard

bold — new result

parameter: solution size



Graph Classes And Their Recognition

Hierarchy of graph classes and the complexity of their respective recognition problem:

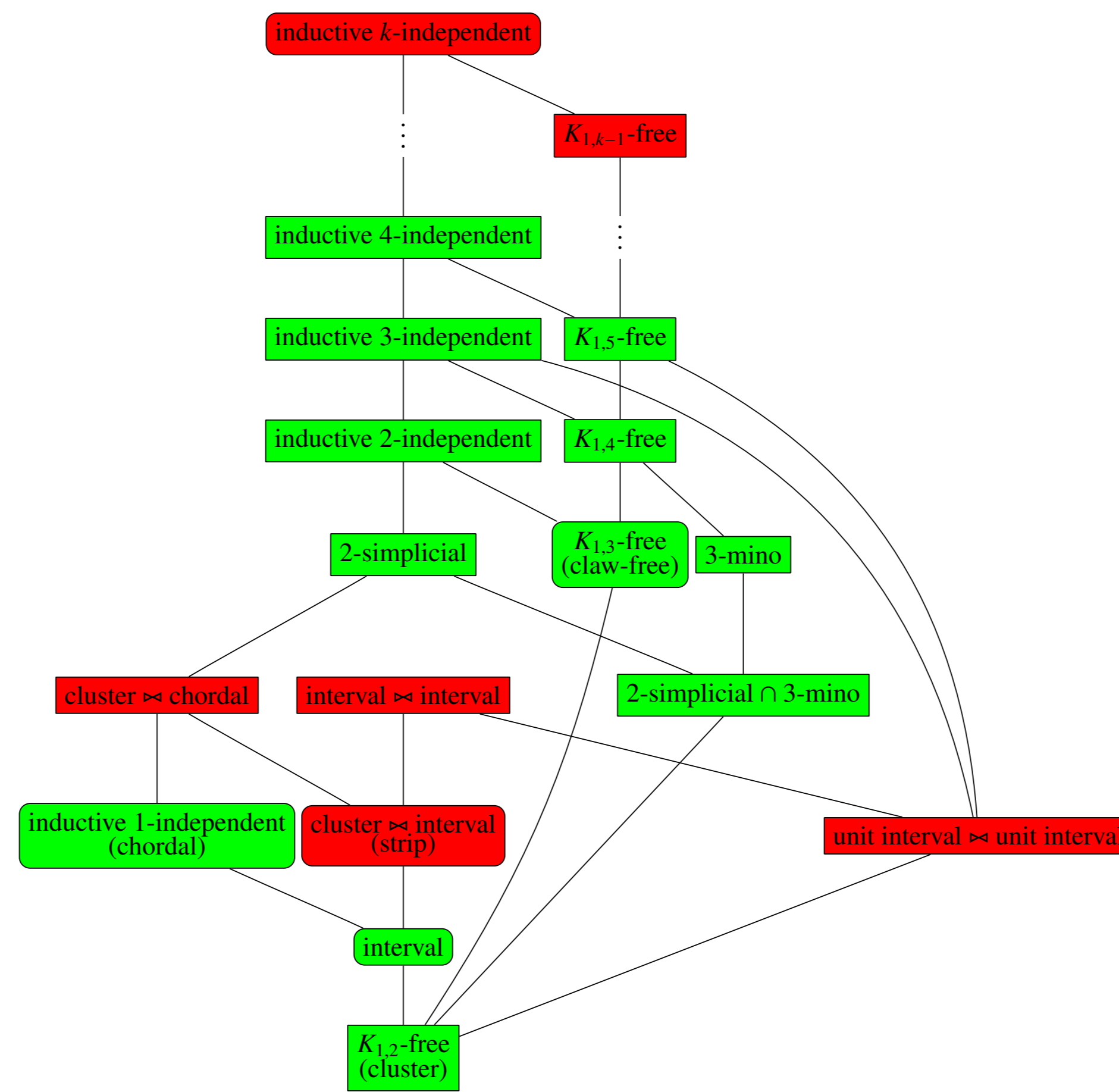
Legend for diagram:

green — polynomial-time recognition

red — recognition NP-hard or W[1]-hard

edge — the lower graph class is properly contained in the upper one

rounded corners — from existing scheduling literature



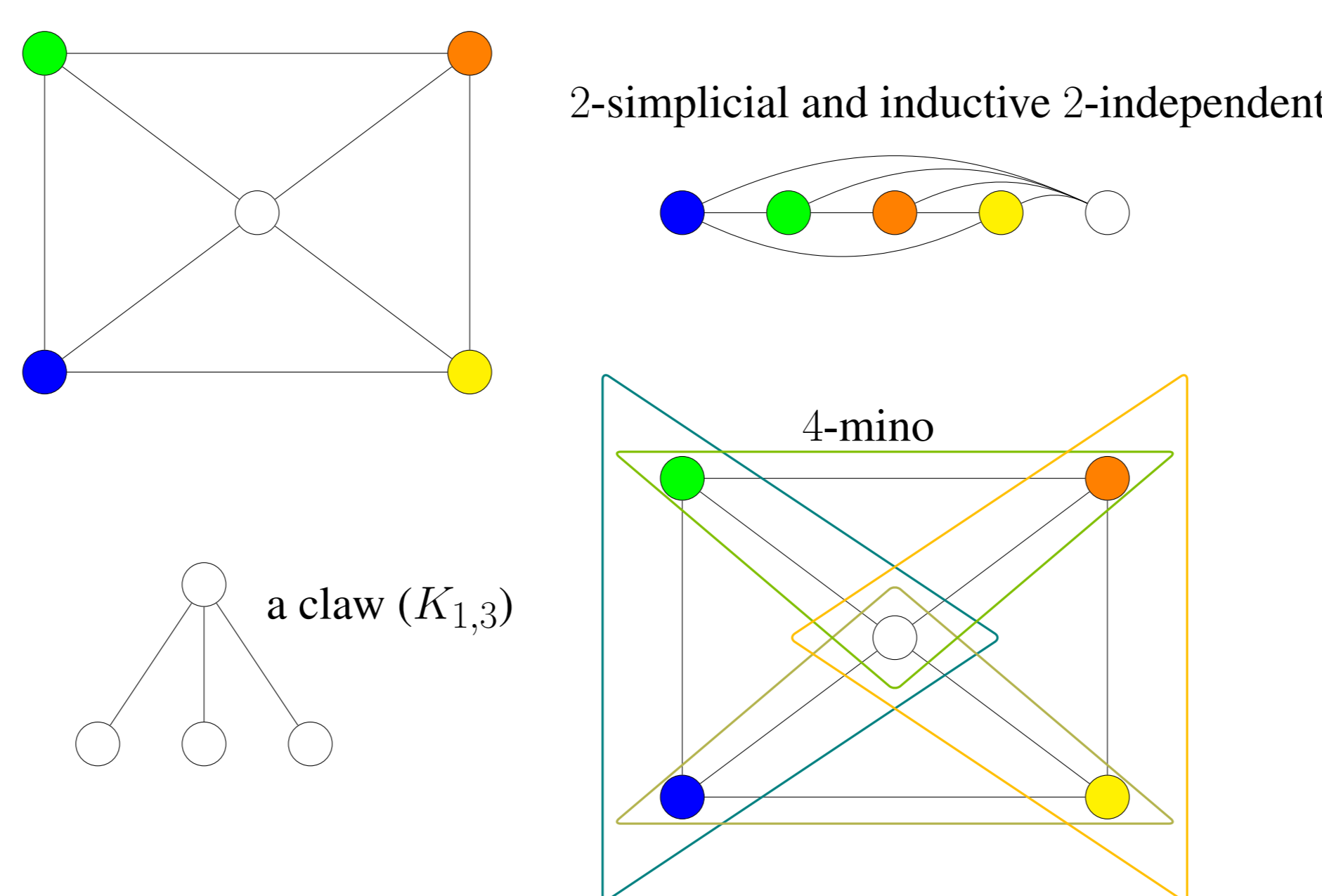
Definition ($A \bowtie B$)

For two graph classes A and B , $A \bowtie B$ is the class of graphs $G = (V, E)$ such that $E = E_1 \cup E_2$ for a graph $G_1 = (V, E_1)$ in class A and a graph $G_2 = (V, E_2)$ in class B .

graph class	definition
k -simplicial	existence of ordering such that “right-neighborhood” can be covered by k cliques
k -mino	each vertex is contained in at most k maximal cliques
$K_{1,k}$ -free	does not contain a $K_{1,k}$ (a tree with one internal node and k leaves) as an induced subgraph
inductive k -independent	existence of ordering such that “right-neighborhood” only contains independent sets of size at most k

$K_{1,3}$ is usually called a claw (\rightsquigarrow claw-free graphs)

Example:



W[1]-Hardness Of MAXIMUM INDEPENDENT SET On 2-Simplicial 3-Minoes

Theorem

MAXIMUM INDEPENDENT SET parameterized by solution size ℓ is W[1]-hard on 2-simplicial 3-minoes.

Idea of Proof: Reduction from MULTICOLORED CLIQUE:

- color gadget for all vertices of a specific color,
- edge gadget for all edges between vertices of different colors.

MULTICOLORED CLIQUE

Input: A graph G whose vertex set is partitioned into independent sets $V_1 \uplus V_2 \uplus \dots \uplus V_k$.

Question: Does G contain a clique of order k ?

Illustration of the reduction from MULTICOLORED CLIQUE to MAXIMUM INDEPENDENT SET:

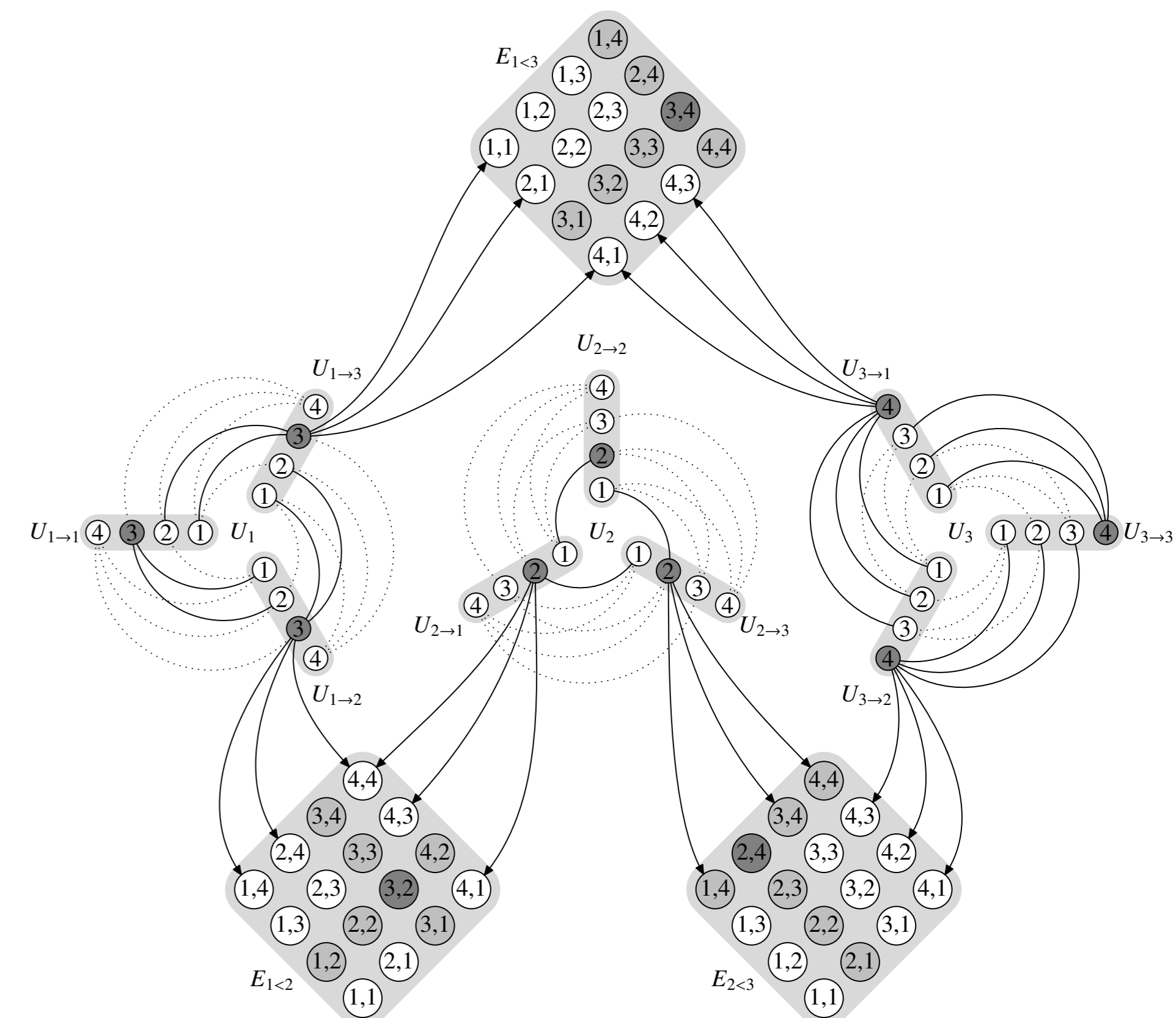
Legend for the picture: Spiral — gadget for all vertices of a color

Grid — gadget for edges between vertices of different colors

Grayshaded areas — cliques

dotted — regular edges (uncluttering the picture)

directed arc — edges between the tail vertex to all vertices in the head column, only shown for dark gray vertices



(three colors of four vertices each)

Challenge in the proof: Show that the resulting graph is 2-simplicial 3-minoe.

Idea to overcome this challenge: Always find a vertex whose neighborhood can be covered by few cliques.

Fixed-Parameter Tractability Of c -COLORABLE SUBGRAPH On Cluster \bowtie Chordal

Theorem

A maximum c -colorable subgraph on at most ℓ vertices of a cluster \bowtie chordal graph can be computed in $2^{\ell+c} \cdot (c+e+3)^\ell \cdot \ell^{O(\log \ell)} \cdot n^2 \cdot \log^3 n$ time if the decomposition of the input graph into a cluster and a chordal graph is given. Herein, e is Euler's number.

Idea of Proof:

- Use MAX-WEIGHT COLORFUL INDEPENDENT SET on chordal graphs as a sub-procedure and solve it using tree decomposition.
- Transfer solutions of MAX-WEIGHT COLORFUL INDEPENDENT SET to solutions of c -COLORABLE SUBGRAPH on cluster \bowtie chordal graphs using color coding two times.

MAX-WEIGHT COLORFUL INDEPENDENT SET

Input: A graph $G = (V, E)$ with vertex weights $w: V \rightarrow \mathbb{N}$ and a coloring $\text{col}: V \rightarrow \{1, 2, \dots, c\}$ of its vertices.

Question: Find a maximum-weight independent set whose vertices have mutually distinct colors.

References

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