Dynamics of Distributed Updating in Fisher Markets

(To Appear in ACM Conference on Economics and Computation (EC) 2018)

(Generalized / Damped Proportional Response Dynamics)

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HALG 2018

Take Home Message

Markets Dynamics to Reach Equilibrium

$\Updownarrow$

Optimization Iterative Processes: Gradient/Mirror Descent

Take Home Message — Markets

JUSTIFY Market Equilibrium (ME)

- ME (and Nash Equilibrium) definition is static — provide no insight on how it can be arrived.

- For special markets, many P-time algorithms. (Ellipsoid Method, Interior Point Method, Flow-Based Combinatorial Algorithm)

- But they cannot be run in real market environment. (Distributed vs. Centralized)

Take Home Message — Markets

JUSTIFY Market Equilibrium (ME)

We need natural algorithm
which can be run
in the highly distributed
market environment.

Take Home Message — Markets

Natural Algorithms / Dynamics:

Tâtonnement (1874): Price Updates (by sellers)

Full Range of Fisher CES Markets

Equivalent to Gradient Descent

(C., Cole, Devanur, STOC 2013)

Proportional Response Dynamics (2007):

(motivated by peer-to-peer network sharing)

Spending Updates (by buyers)

Substitute Range of Fisher CES Markets

(Wu, Zhang STOC 2007; Zhang ICALP 2009)

Take Home Message — Markets

Proportional Response Dynamics (PRD):

Spending Updates (by buyers)

Substitute Range of Fisher CES Markets

OPEN: What about Complementary / Mixed ranges?

By establishing

equivalence to Mirror Descent,

we derive new form of PRD

which converges to ME in the

complementary /mixed ranges.

Take Home Message — Markets

For Complementary Range:

$$b_{ij}(t+1) \;\leftarrow\; \frac{a_{ij}\cdot(\text{total spending on good } j \text{ at time } t)^{\rho/(\rho-1)}}{\sum_k a_{ik}\cdot(\text{total spending on good } k \text{ at time } t)^{\rho/(\rho-1)}}$$

Recall that for Substitute Range:

$$b_{ij}(t+1) \;\leftarrow\; \frac{a_{ij}\cdot(\text{quantity of good } j \text{ obtained at time } t)^{\rho}}{\sum_k a_{ik}\cdot(\text{quantity of good } k \text{ obtained at time } t)^{\rho}}$$

Take Home Message — Optimization

Proportional Response Dynamics (PRD)

$\equiv$ Mirror Descent (MD)

PRD Converges to ME

$\equiv$ MD Converges to Minimum / Saddle Point

TECHNICAL HIGHLIGHTS:

We present new MD analyses to deal with

new and broad classes of functions.

(Bregman Convex or Convex-Concave Functions)

Take Home Message — Optimization

Convex-Concave Function:

$f(\mathbf{x}, \mathbf{y})$, convex in $\mathbf{x}$, concave in $\mathbf{y}$

$(\sigma_X, \sigma_Y, L_X, L_Y)$**-Strongly Bregman Convex-Concave Function:**

$$
\begin{aligned}
&- L_Y \cdot d_h(\mathbf{y}', \mathbf{y}) + \sigma_X \cdot d_g(\mathbf{x}', \mathbf{x}) \\
&\quad \le f(\mathbf{x}', \mathbf{y}') - \text{linearization at } f(\mathbf{x}, \mathbf{y}) \\
&\qquad \le -\sigma_Y \cdot d_h(\mathbf{y}', \mathbf{y}) + L_X \cdot d_g(\mathbf{x}', \mathbf{x}).
\end{aligned}
$$

d_g, d_h are ANY Bregman divergences.

Take Home Message — Optimization

$(\sigma_X, \sigma_Y, L_X, L_Y)$-**Strongly Bregman Convex-Concave Function:**

$$
\begin{aligned}
&- L_Y \cdot d_h(\mathbf{y}', \mathbf{y}) + \sigma_X \cdot d_g(\mathbf{x}', \mathbf{x}) \\
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&\quad\quad \leq -\sigma_Y \cdot d_h(\mathbf{y}', \mathbf{y}) + L_X \cdot d_g(\mathbf{x}', \mathbf{x}).
\end{aligned}
$$

Update Rule:

$$\mathbf{x}^{t+1} \leftarrow \arg\min_{\mathbf{x}} \left\{ \left\langle \nabla_{\mathbf{x}} f(\mathbf{x}^t, \mathbf{y}^t) \ , \ \mathbf{x} - \mathbf{x}^t \right\rangle \ + \ \mathbf{2} L_X \cdot d_g(\mathbf{x}, \mathbf{x}^t) \right\}$$

$$\mathbf{y}^{t+1} \leftarrow \arg\min_{\mathbf{y}} \left\{ - \left\langle \nabla_{\mathbf{y}} f(\mathbf{x}^t, \mathbf{y}^t) \ , \ \mathbf{y} - \mathbf{y}^t \right\rangle \ + \ \mathbf{2} L_Y \cdot d_h(\mathbf{y}, \mathbf{y}^t) \right\}$$

THEOREM: (by a really clean analysis)

Pointwise Convergence to Saddle Point; Linear Convergence Rate.

MORE DETAILS AT POSTER

19th Max Planck Advanced Course on the Foundations of Computer Science

13 - 17 August 2018, Saarbrücken, Germany

Fine-Grained Complexity and Algorithms

Ramamohan Paturi

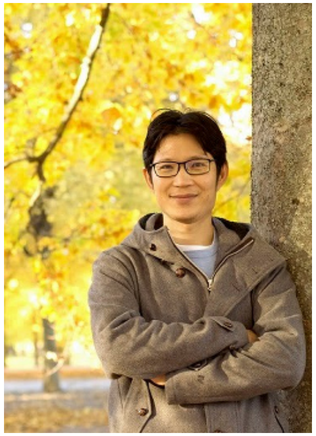
UC San Diego

Foundations of Fine-grained Complexity

Amir Abboud

IBM Almaden

Hardness in P



Danupon Nanongkai

KTH

Dynamic graphs: algorithms, conditional lower bounds, and complexity classes

google „ADFOCS“

max planck institut informatik