On the Complexity of Closest Pair via Polar-Pair of Point-Sets

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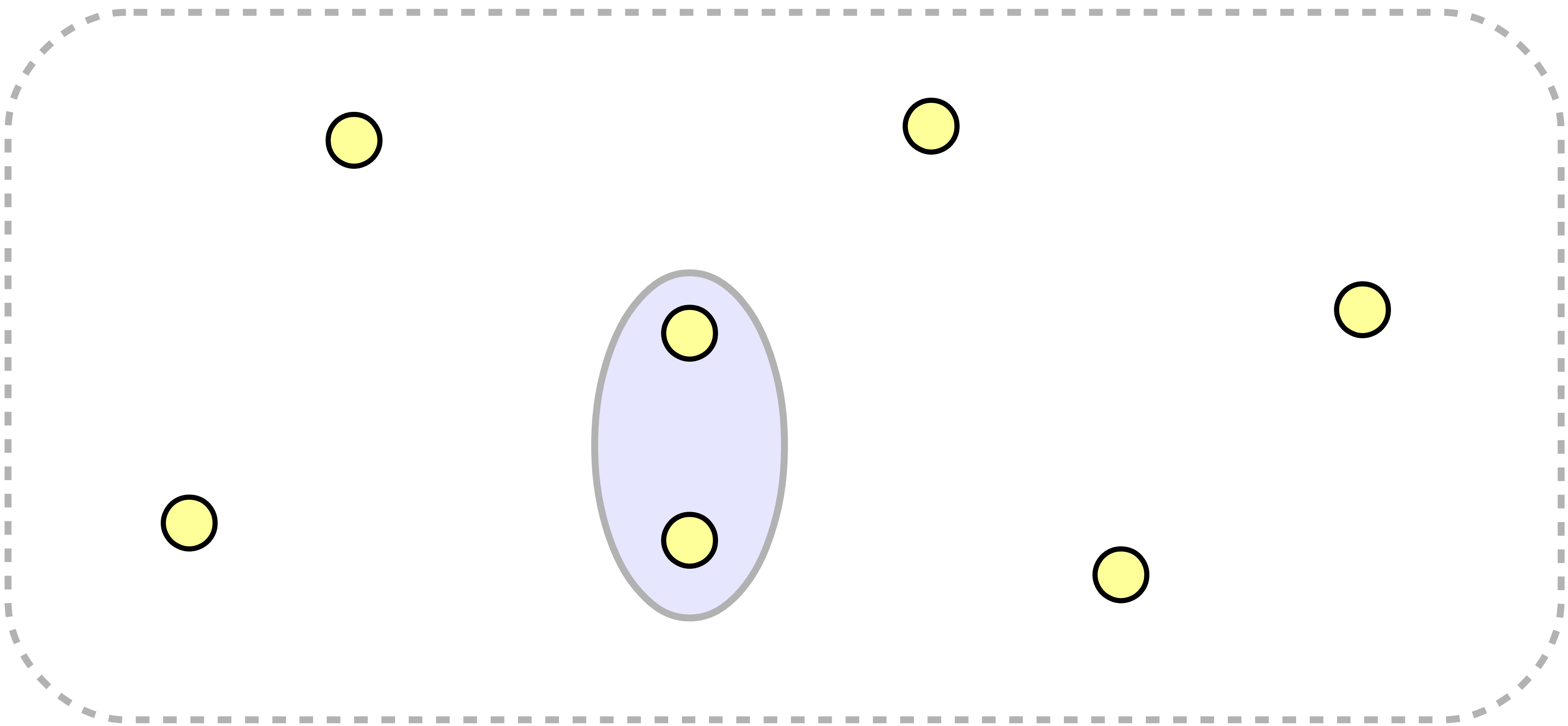
Joint work with Roee David and Karthik CS

This Talk

- Complexity of Closest Pair
- Geometric Representation of Graphs

Closest Pair (CP)

Given a collection of n points in a d-dimensional metric, find a pair of points with minimum-distance.



Known Algorithms

- Euclidean Closest Pair
  - Dimension d=O(1):
    - $O(2^D n \log n)$ (deterministic) [Bently-Shamos'76]
    - $O(2^D n)$ (randomized) [Rabin'76, Khuller-Mattias'95]
  - Dimension $d=\Theta(\log n)$
    - $O(d n^2)$ (trivial algorithm)
  - Dimension d=n: $O(n^{3-\varepsilon})$, for some $\varepsilon > 0$

Is there an $O(n^{1.9})$-time algorithm when dimension $d=\Omega(\log n)$?

Is there an $O(n^{1.9})$-time algorithm when dimension $d=\Omega(\log n)$?

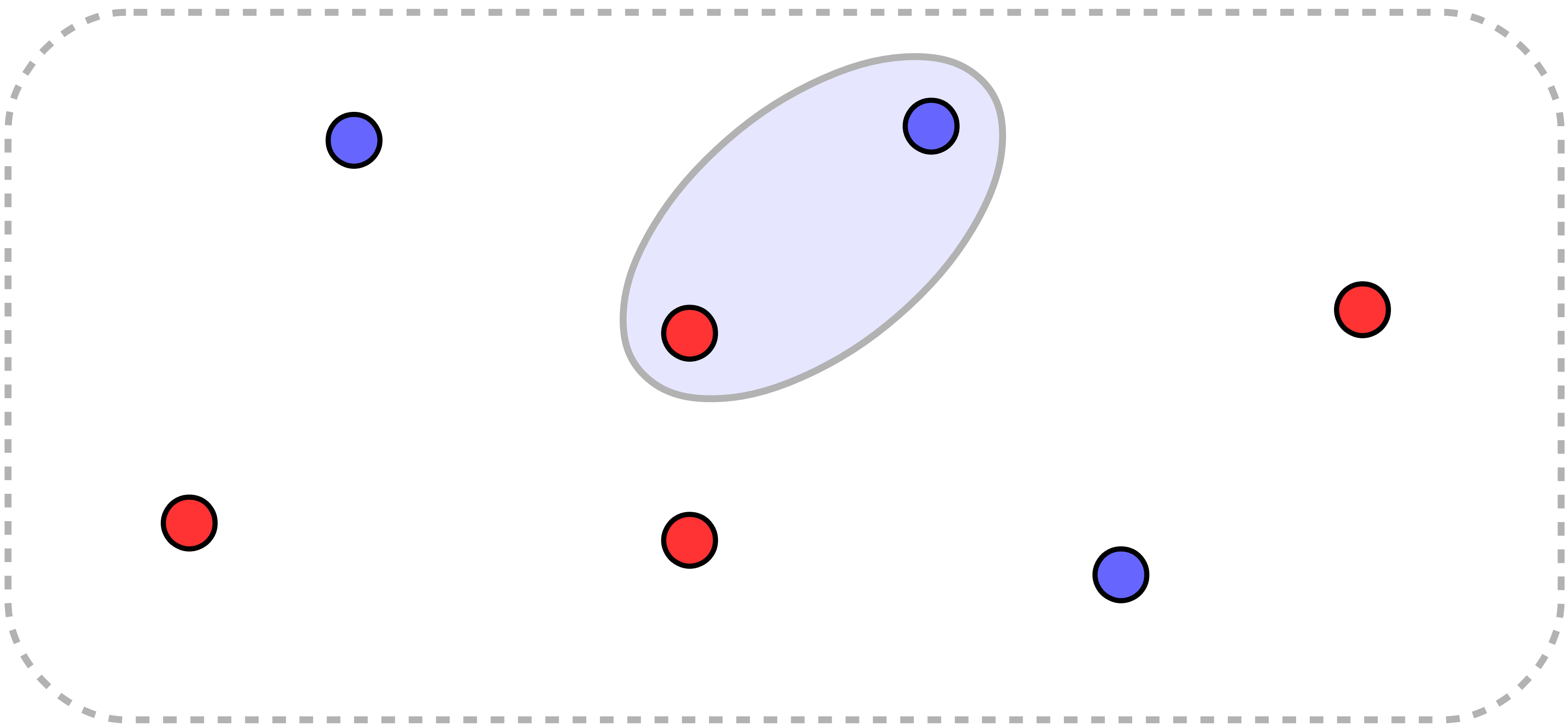
Don't know for Euclidean Closest Pair.

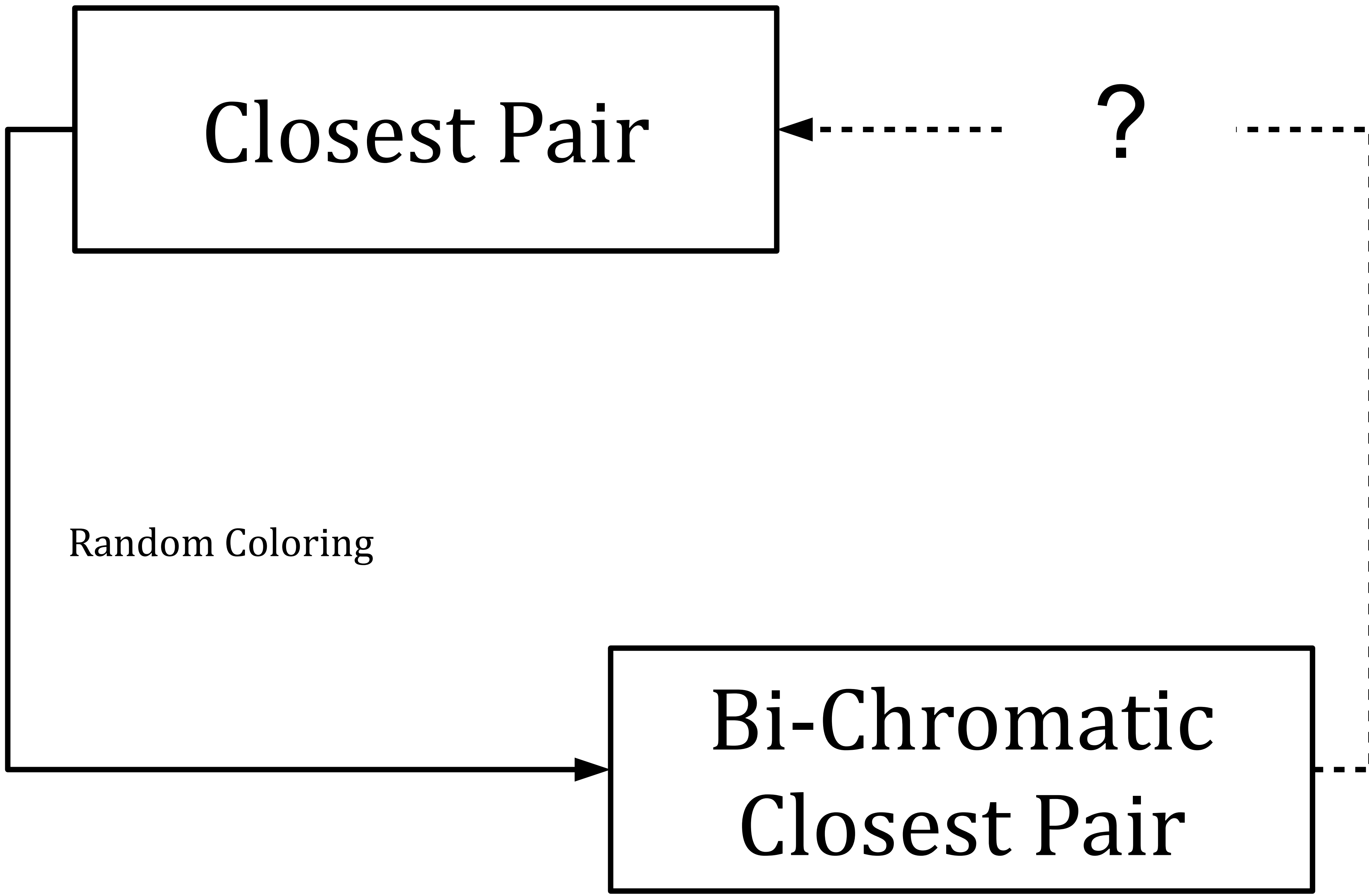
No for the bichromatic variant.

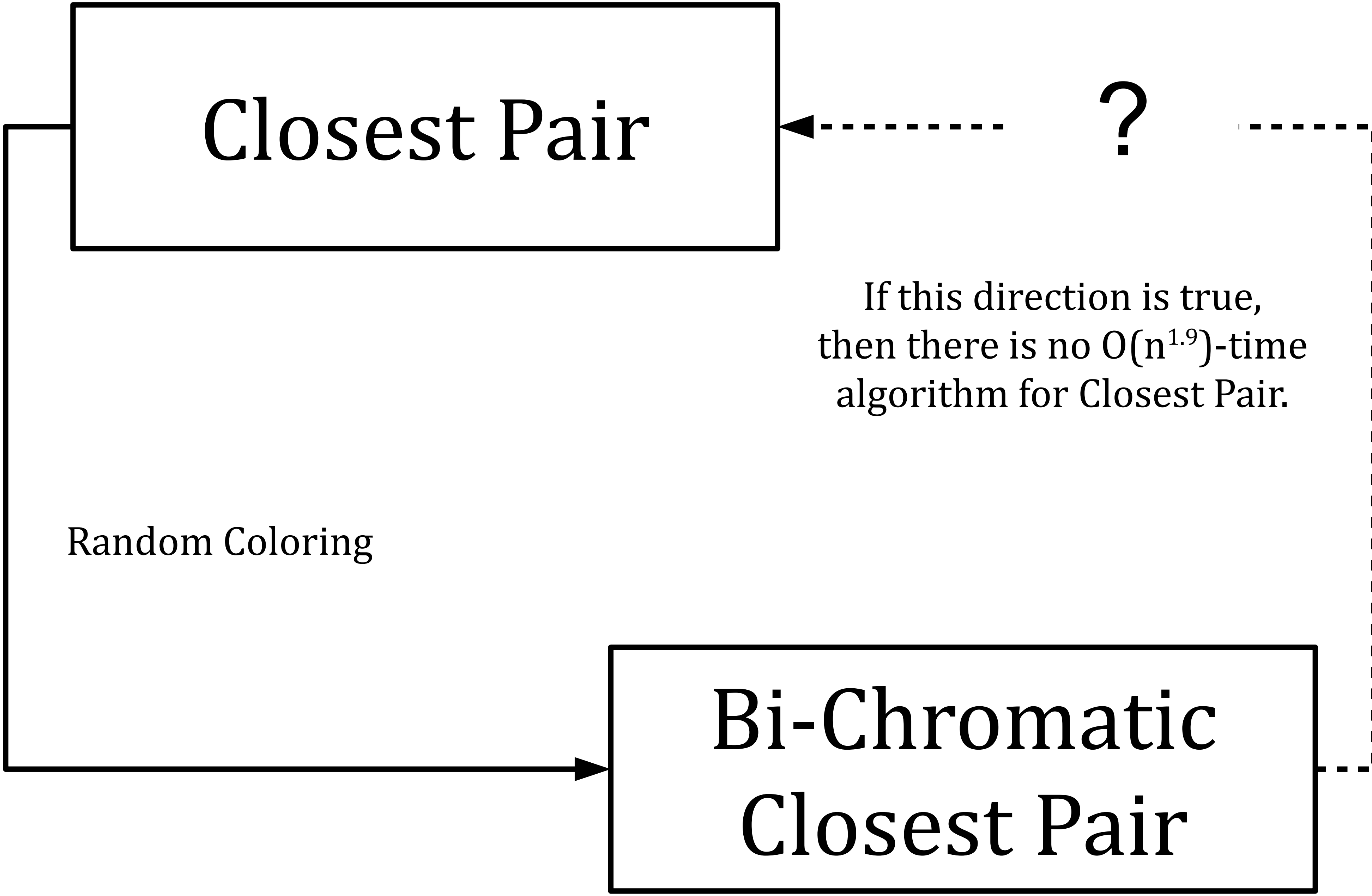
[Alman-Williams 2015]

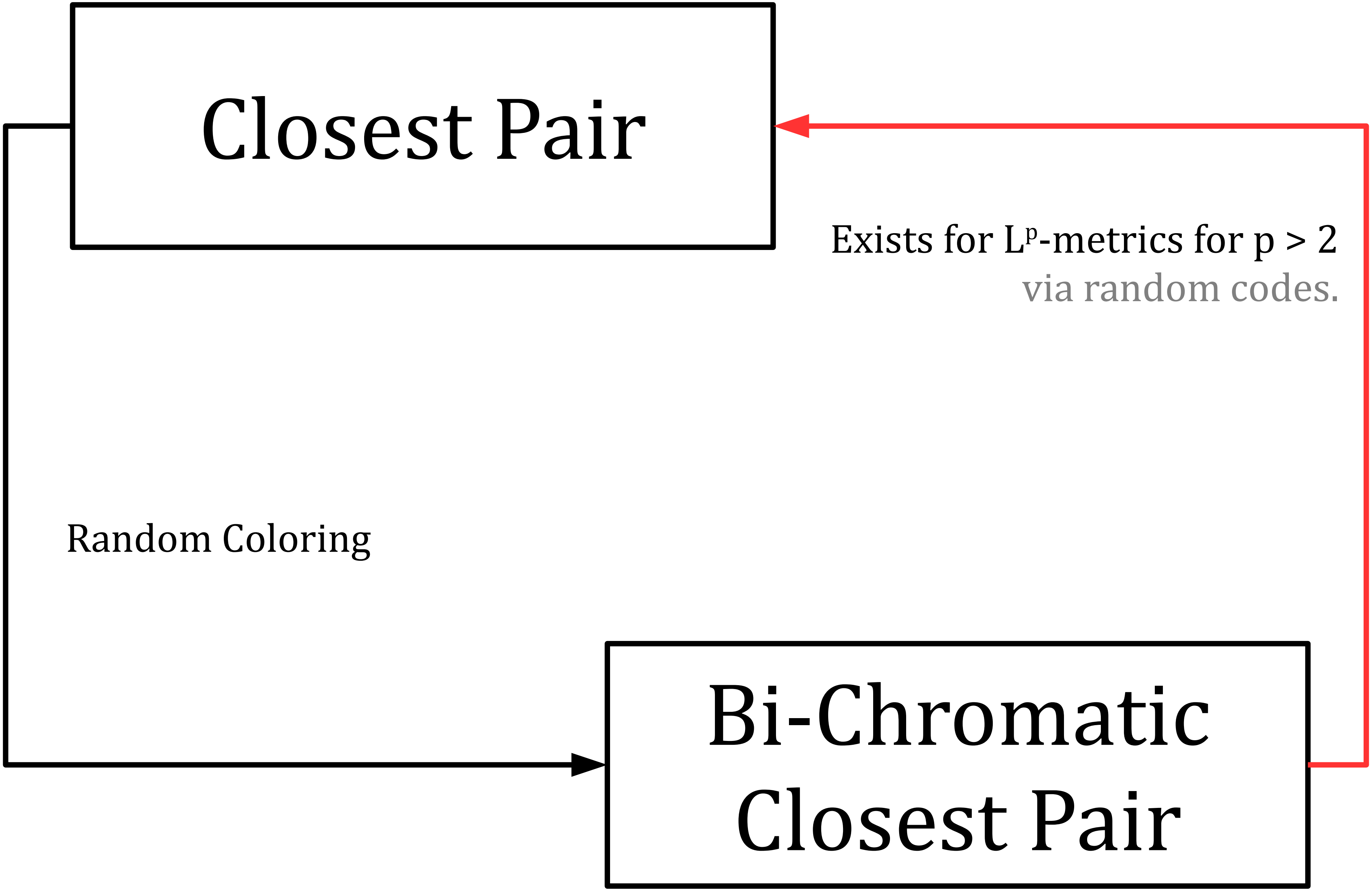
Bi-Chromatic Closest Pair (BCP)

Given a collection of n red and n blue points in a d-dimensional metric, find a pair of red-blue points with minimum-distance.



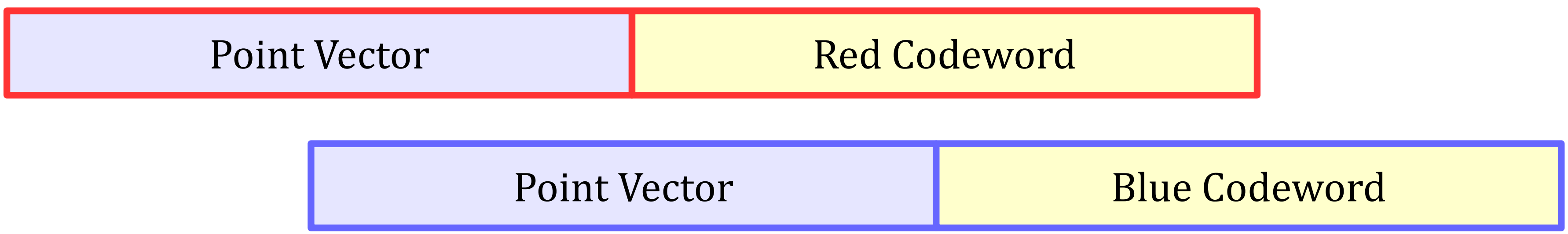

Closest Pair
?
Random Coloring
Bi-Chromatic
Closest Pair


Closest Pair
?
If this direction is true,
then there is no $O(n^{1.9})$-time
algorithm for Closest Pair.
Random Coloring
Bi-Chromatic
Closest Pair


Closest Pair
Exists for L^p-metrics for $p > 2$
via random codes.
Random Coloring
Bi-Chromatic Closest Pair

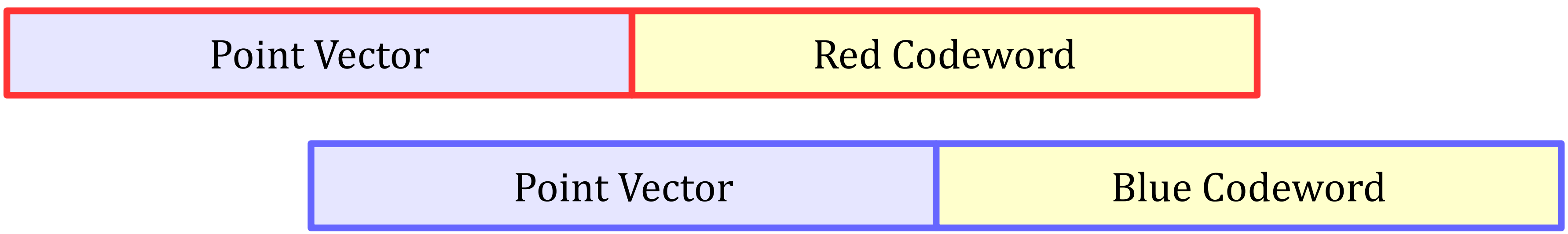
Reduction BCP → CP

Concatenate point-vectors with codewords



Reduction BCP → CP

Concatenate point-vectors with codewords



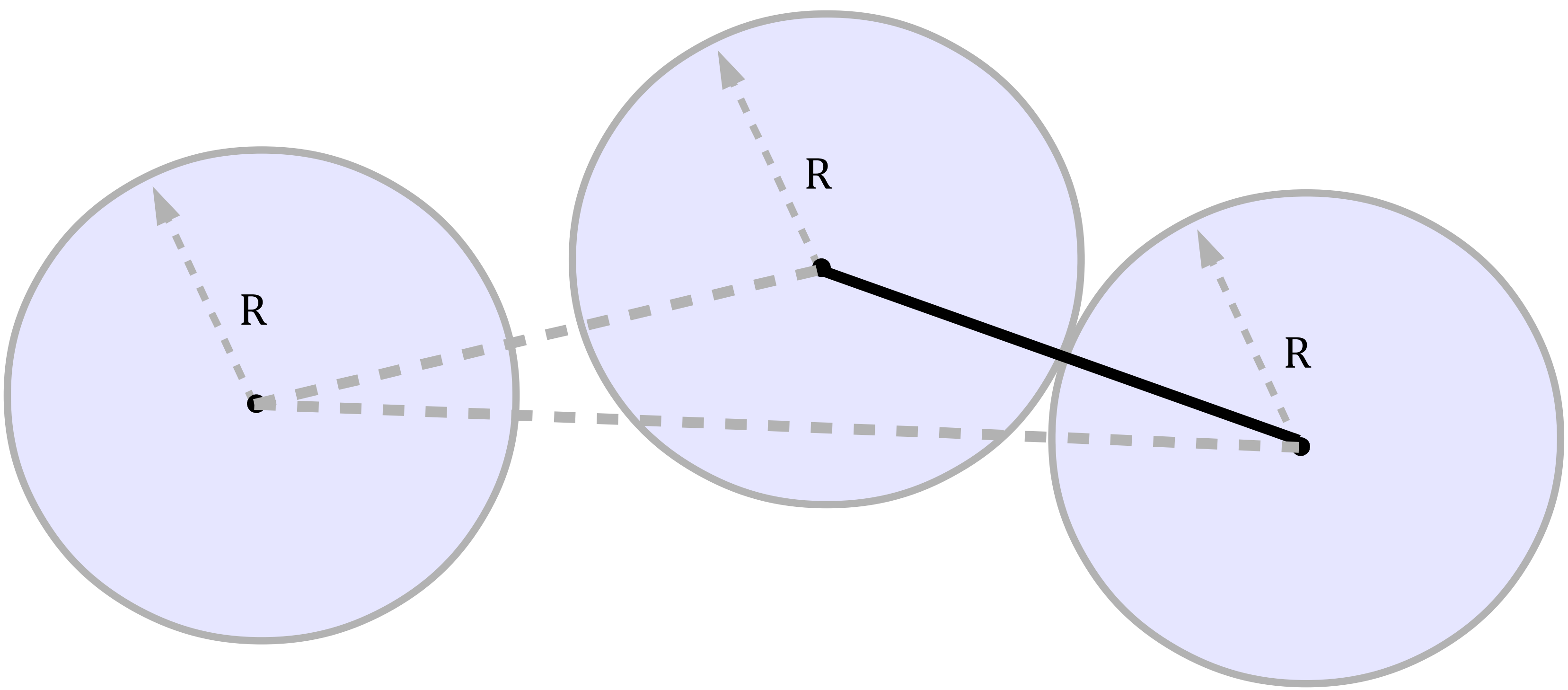
Needed Properties of The Codewords
(Bi-Clique Property)

Distance(Red-Code, Red-Code') $\geq R + 1/n$
Distance(Blue-Code, Blue-Code') $\geq R + 1/n$
Distance(Red-Code, Blue-Code) $= R$

The existence of Codewords with Bi-Clique Property implies BCP → CP

(that runs in $O(n^{1.9})$-time)

Complexity Question of CP reduces to Geometric Representation of Bi-Clique



Complexity Question of CP reduces to Geometric Representation of Bi-Clique

What is the smallest dimension to represent a bi-clique in L^p-metric?

(contact-dimension of bi-clique (bicd))

CP & BCP are equivalent in $O(\log n)$-dimension L^p-metrics (for $p>2$)

CP & BCP are equivalent in O(log n)-dimension L^p-metrics (for p>2)

How about other L^p-metrics?

Known Bounds

for Bi-Clique Contact Dimension

	n	$\leq \mathrm{bicd}(L^0)$	$\leq n$	
	?	$\leq \mathrm{bicd}(L^1)$	$\leq n^2$	
	?	$\leq \mathrm{bicd}(L^p)$	$\leq n$	for $1 < p < 2$
Maehara 1985 Frankl-Maehara 1988	$1.286\,n$	$\leq \mathrm{bicd}(L^2)$	$\leq 1.5\,n$	
	$\Omega(\log n)$	$\leq \mathrm{bicd}(L^p)$	$\leq O(\log n)$	for $p > 2$
	$\Omega(\log n)$	$\leq \mathrm{bicd}(L^\infty)$	$\leq 2 \log_2 n$	

Known Bounds

for Bi-Clique Contact Dimension

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$1.286\, n$	$\leq \mathrm{bicd}(L^2)$	$\leq 1.5\, n$	
$\Omega(\log n)$	$\leq \mathrm{bicd}(L^p)$	$\leq O(\log n)$	for $p > 2$
$\Omega(\log n)$	$\leq \mathrm{bicd}(L^\infty)$	$\leq 2 \log_2 n$	

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Bi-Clique has no contact-graph
in $O(\log n)$-dimension for most L^p-metrics.

Open Problems

- What is the lower bound for bicd(L^1) ?
  - Related to Kusner's conjecture on equilateral dimension of L^1
    - For clique, Alon-Pavel shows contact-dim $\geq \Omega(n / \log n)$
- Better Lower / Upper Bounds for L^1 and L^2 ?
- An alternative way to reduce BCP → CP ?
  - One will need a white-box reduction.

The End

Thank you for your attention.

Questions?

The End

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Questions?

Karthik C.S. will give a 20 min talk in SoCG'18 next week.