Improved Algorithm for Weighted Flow Time

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Joint work with Yossi Azar

Overview

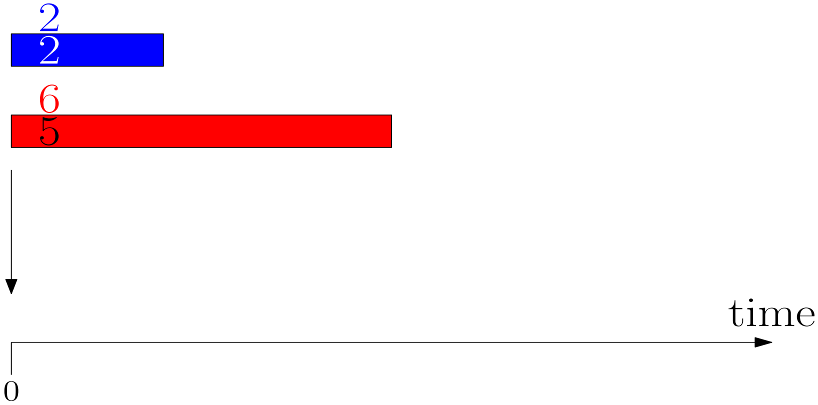
- Single machine receives jobs over time.
- Each job has an arrival time, a weight and processing time (volume).
- Allow preemption.

Overview

- Goal function: minimize the *weighted flow time* (weighted response time):

$$\sum_{\text{job } J} w(J) \cdot (c(J) - r(J))$$

Overview



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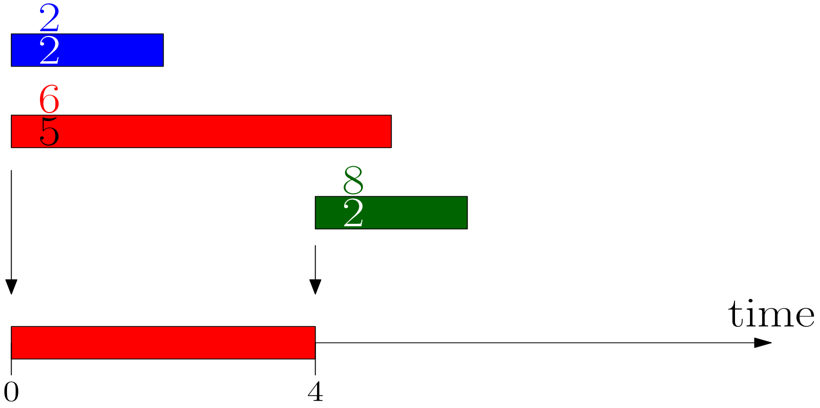

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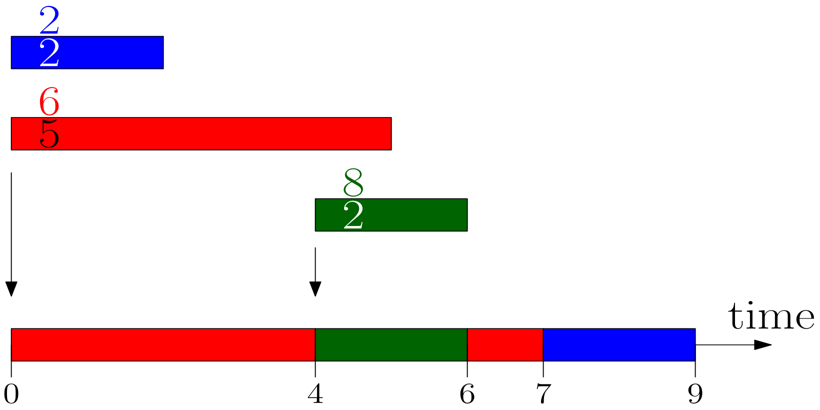

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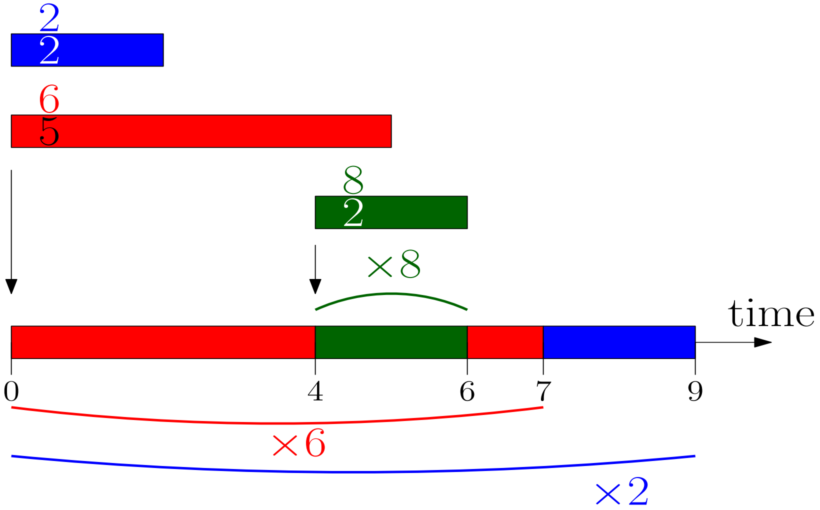

Unweighted Case

- Unit weights $\Longrightarrow$ regular flow-time.
- SRPT is optimal.

Weighted Case – Highest Density First

- Density – ratio of weight to processing time.
- HDF: Natural generalization of SRPT for weights.
- HDF is bad!

Previous Work– Weighted Flow Time on Single Machine

- $O(\log^2 P)$-competitive alg. [ChekuriKhannaZhu, STOC '01]
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- Non-constant lower bound [BansalChan, SODA '09]

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- A $O(\log(\min(W, P, D)))$-competitive algorithm.
    - W, P, D are unknown in advance.

Our $O(\log P)$ Algorithm

- We now describe our $O(\log P)$-competitive algorithm.

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- At any time t, process top job from bin i.
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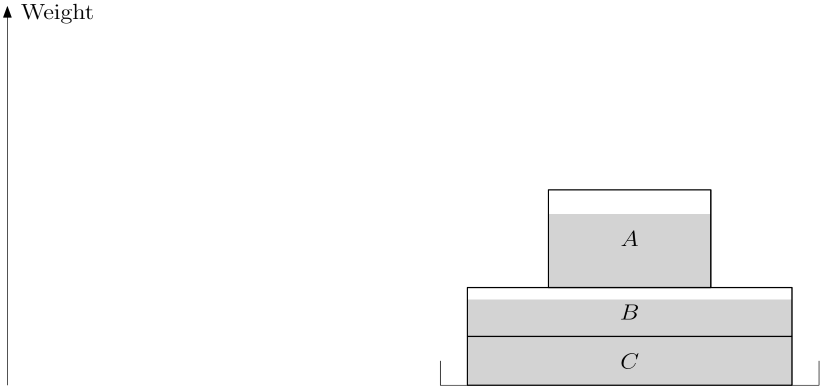
  - **OR** – is top job's volume at most 2^i?

Illustration – Job Arrival



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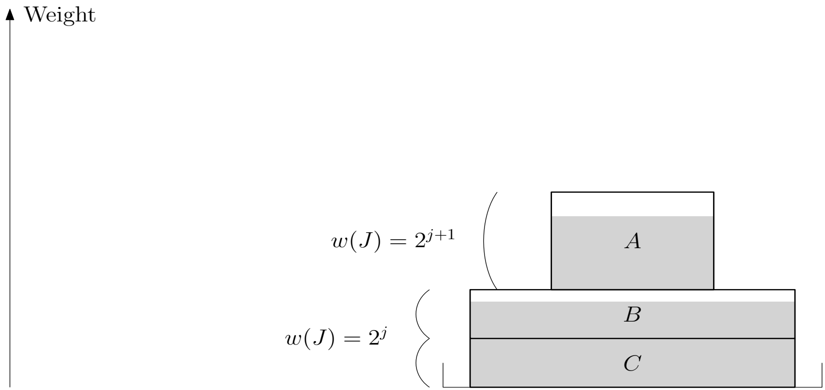

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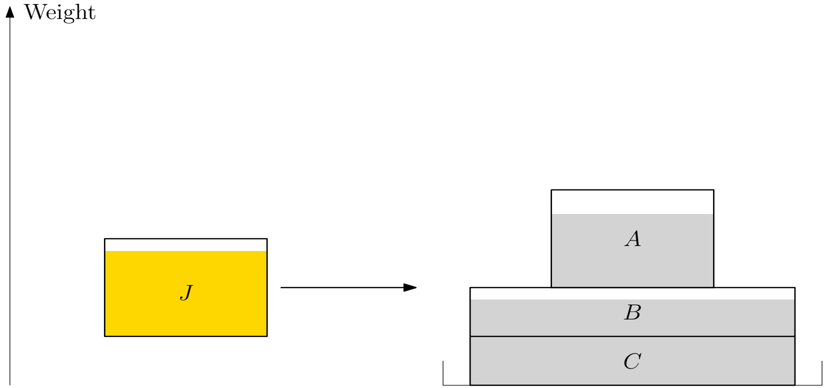

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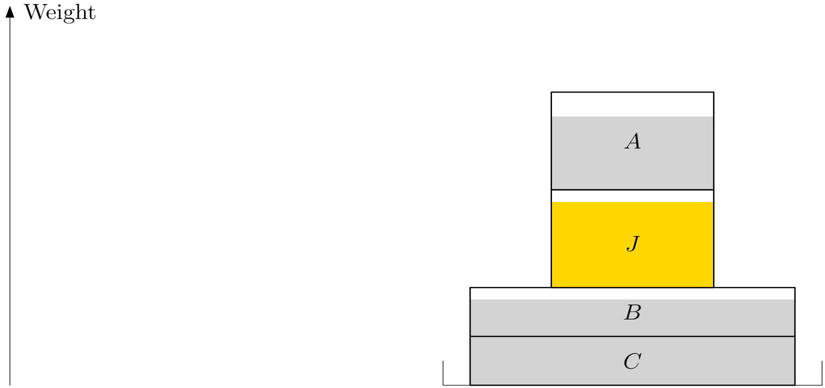

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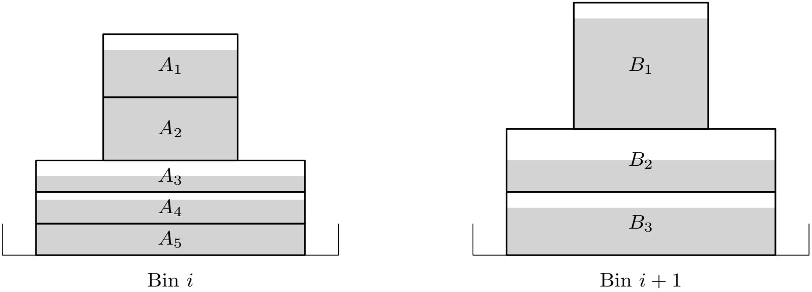

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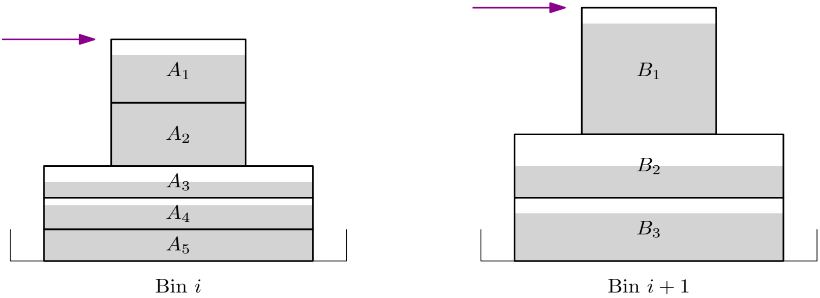

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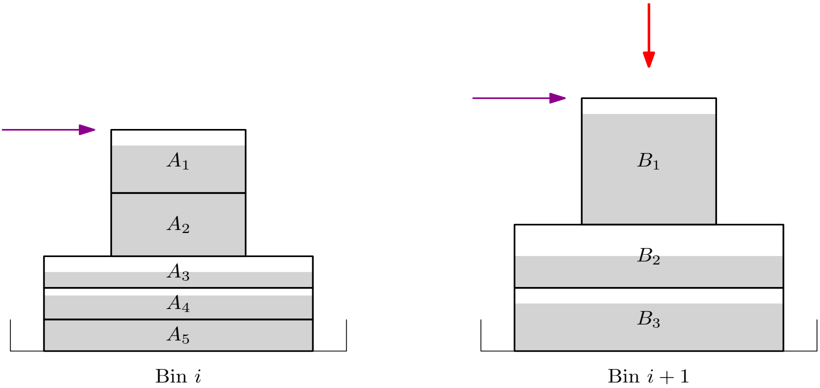

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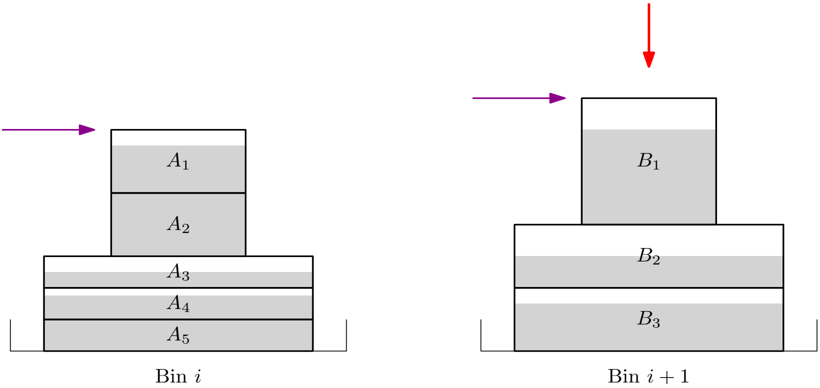

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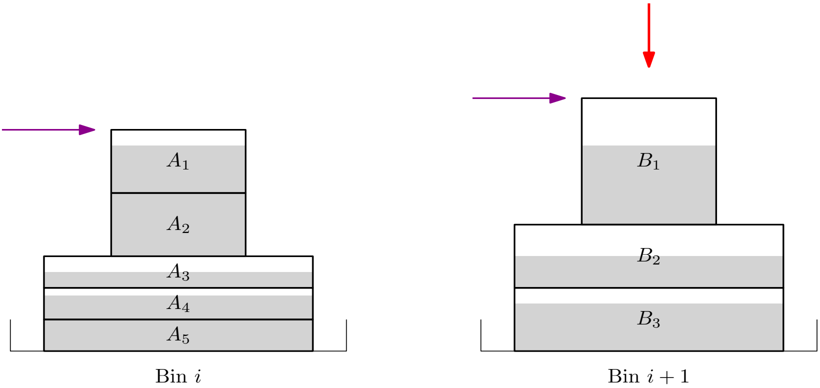

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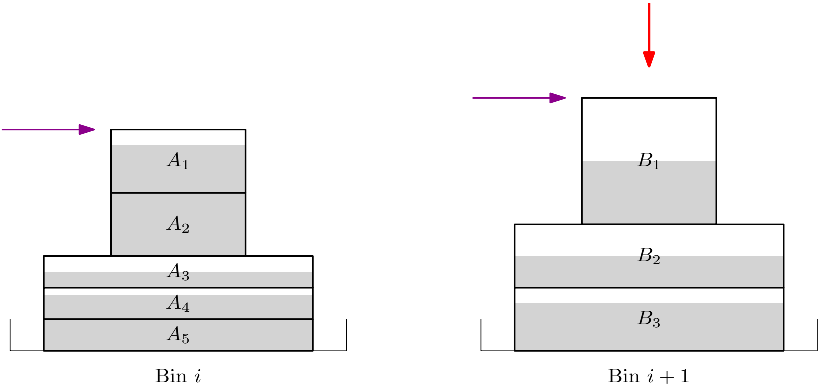

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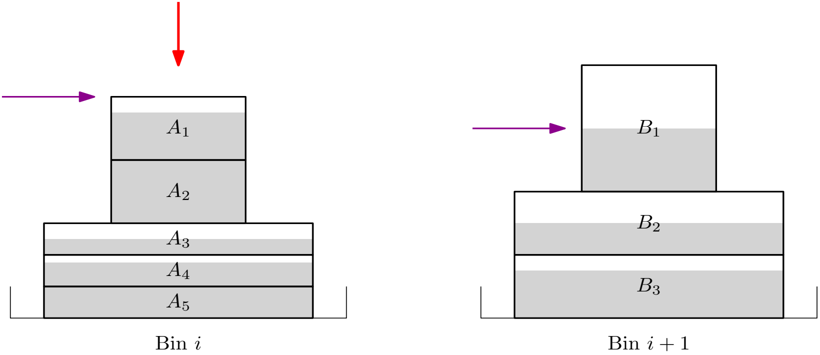
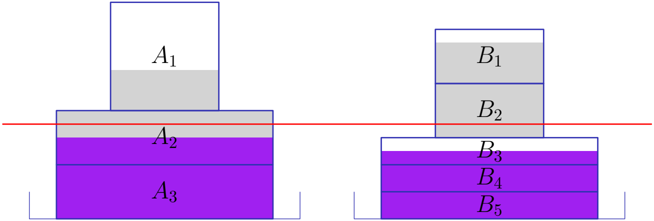


Analysis – Covered Volume

- Place a horizontal bar at some height (weight).
- The bar covers some volume.



Conclusion

- Algorithms for weighted flow time on a single machine:
  - $O(\log P)$ competitive.
  - $O(\log D)$ competitive.
  - $O(\log(\min(P, W, D)))$ competitive.
- Open problem: an exponential gap between $O(\log P)$ and the lower bound of $\Omega(\sqrt{\frac{\log \log P}{\log \log \log P}})$.