Towards Hybrid Programmable Matter: shape recognition, formation, and sealing algorithms for finite automaton robots

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formation

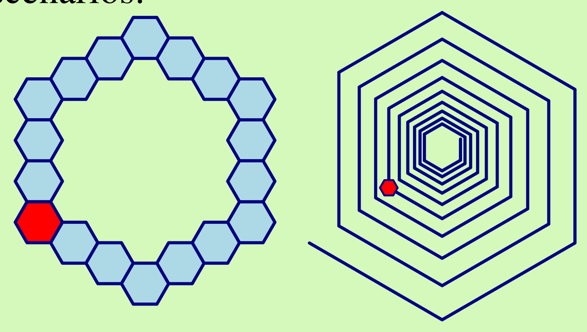
Problem statement: Given a shape (e.g., a triangle), transform any initial set of tiles into this given shape. Refer to [1, 2].

Observation 1. From a given set of tiles it is not always possible to find a tile that does not disconnect the tile set. Therefore, the approach to start removing tiles and building a required shape is not always going to work. We need an intermediate step!

Observation 2. If a robot can always find a safely removable tile, it can build simple shapes layer by layer.

Theorem 1. A single robot cannot find a safely removable tile.

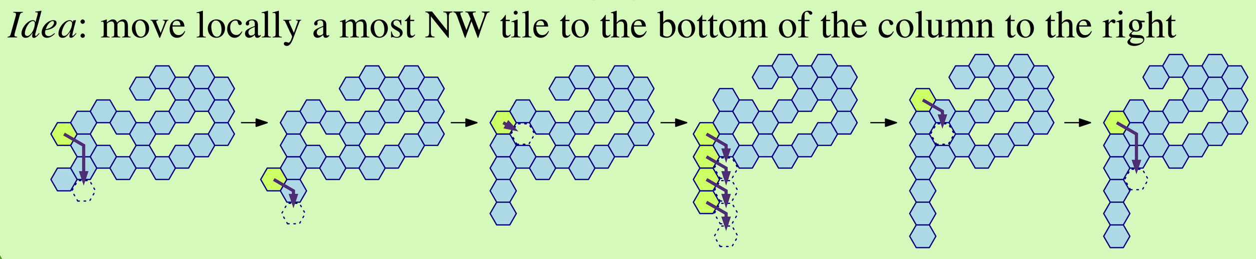
Proof idea. It cannot distinguish between these two scenarios:



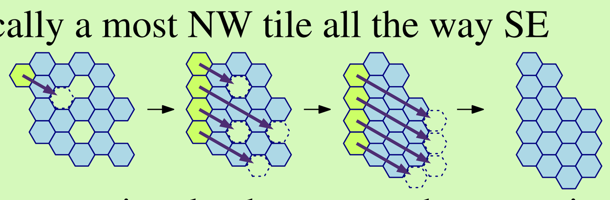
Theorem 2. A robot with a pebble can always find a safely removable tile.

Intermediate structures: before assembling the final shape, the robots reshape the tiles to form a *simply connected* set. We describe three structures:

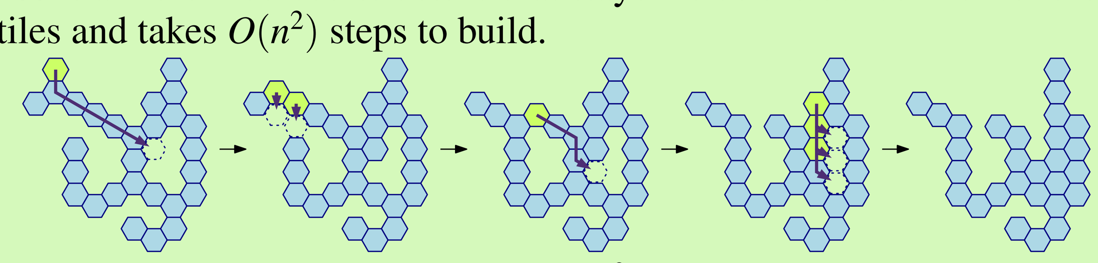
- **Line** is a simple structure that takes $O(n^2)$ steps to build.
  Idea: move locally a most NW tile to the bottom of the column to the right



- **Block** has at most the diameter of the input tile set, can be built in $O(nD)$ steps.
  Idea: move locally a most NW tile all the way SE



- **Tree** is a more involved structure that stays inside the convex hull of the input tiles and takes $O(n^2)$ steps to build.



Theorem 3. A *line* can be built in $O(n^2)$ steps, a *block* in $O(nD)$ steps (D is the diameter of the tile set), a *tree* can be built in $O(n^2)$ steps.

[1] R. Gmyr, I. Kostitsyna, F. Kuhn, C. Scheideler, and T. Strothmann. Forming tile shapes with a single robot. In *Abstr. European Workshop on Computational Geometry (EuroCG)*, 2017

[2] R. Gmyr, K. Hinnenthal, I. Kostitsyna, F. Kuhn, D. Rudolph, C. Scheideler, and T. Strothmann. Forming Tile Shapes With Simple Robots. To appear in *Abstr. 6th Workshop on Biological Distributed Algorithms*, 2018

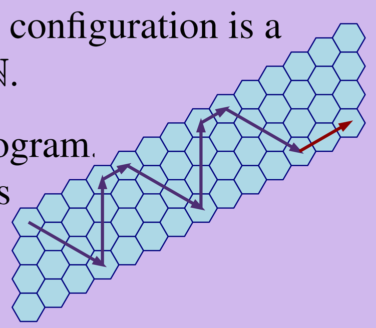
recognition

Problem statement: Given a set of input tiles and a function $f(\cdot)$, check if the tiles form a parallelogram with sides $h \times \ell$, where $\ell = f(h)$.

Robot with no pebbles.

Theorem 1. A single robot can detect whether the tile configuration is a parallelogram with $\ell = ah + b$ for any constants $a, b \in \mathbb{N}$.

Proof idea: First, verify that the configuration is a parallelogram. Then, starting at the most NW tile, move along SE-N zig-zags counting up to a, and finish by making b steps NE. Verify that the end position is the most SE.



Theorem 2. A single robot without any pebbles cannot decide whether the tile configuration is a parallelogram with $\ell = f(h)$, where $f(x) = \omega(x)$.

Robot with one pebble.

Theorem 3. A robot with one pebble can decide whether the tile configuration is a parallelogram with length $\ell = p(h)$, where h is its height, for any given polynomial $p(\cdot)$ of constant degree n.

Proof idea: We represent the polynomial as

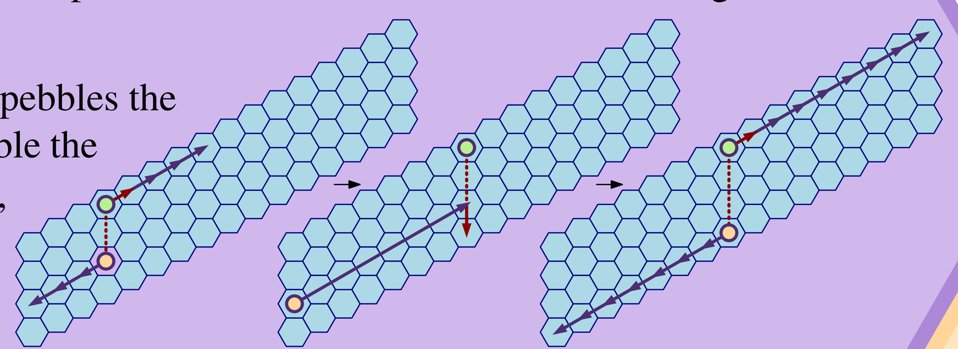
$$p(x) = a_n \cdot (x)_n + a_{n-1} \cdot (x)_{n-1} + \ldots + a_1 \cdot x + a_0,$$

where $(x)_i := x!/(x-i)!$ and a_is are constants. The robot will move the pebble from the leftmost to the rightmost column in phases, by $|a_i(h)_i|$ steps in each phase. We use the fact that the function $\frac{(x)_i}{\mathrm{lcm}(x,\ldots,x-i+1)}$ is periodic with period $\mathrm{lcm}(1,\ldots,i-1)$, which is a constant. By combining zigzag movement with checking local neighborhoods, the robot can shift the pebble by $\mathrm{lcm}(h,\ldots,h-i+1)$ for each i, which can be combined to a shift by $|a_i(h)_i|$ steps.

Robot with two pebbles.

Theorem 4. A robot with two pebbles can decide whether the tile configuration is parallelogram with $\ell = 2^h$.

Proof idea: By using two pebbles the robot can iteratively double the distance between them, thus computing the powers of 2.



Theorem 4. A robot with two pebbles can decide whether a tile configuration is a parallelogram with $\ell = 2^{2^{\cdot^{\cdot^{h}}}}$, where the power tower is of constant height.

[1] R. Gmyr, K. Hinnenthal, I. Kostitsyna, F. Kuhn, D. Rudolph, and C. Scheideler. Shape Recognition by a Finite Automaton Robot. In *Abstr. European Workshop on Computational Geometry (EuroCG)*, 2018

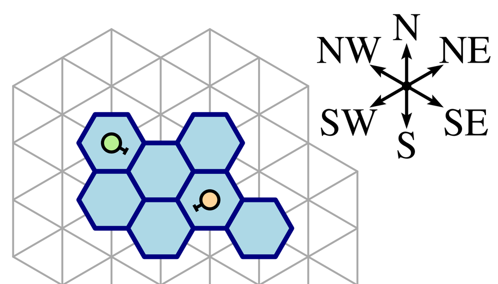
model

Motivated by the problem of manipulating nanoscale materials by nanoscale active agents, we introduce a new theoretical hybrid model for programmable matter.

Tiles: passive agents

Robots: active agents

Robots act as *deterministic finite automata* and operate in *look-compute-move* cycles.



In the *look* phase each robot can observe its local neighborhood.

In the *compute* phase a robot can change its state and determine its next move.

In the *move* phase a robot can either

(1) pick up a tile,
(2) place a tile it is carrying, or
(3) move to an adjacent node while possibly carrying a tile.

In this project we have considered three types of problems:

- shape formation,
- shape recognition, and
- sealing.

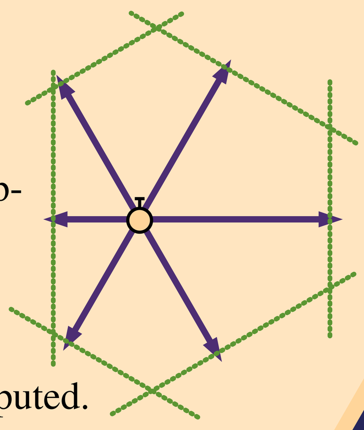
sealing

Refer to the full paper for details [1].

Problem statement: Given a set of robots and an object, enclose it in a shortest cycle, i.e., a convex hull.

Model extention.
We extend the robot model with a capability of expansion and contraction, analogous to [2].

Idea: A leader can maintain 6 distances to the boundary of the convex hull as counters.



Walking along the boundary of the object, the leader will update the values of the counters when stepping outside of so far computed convex hull.

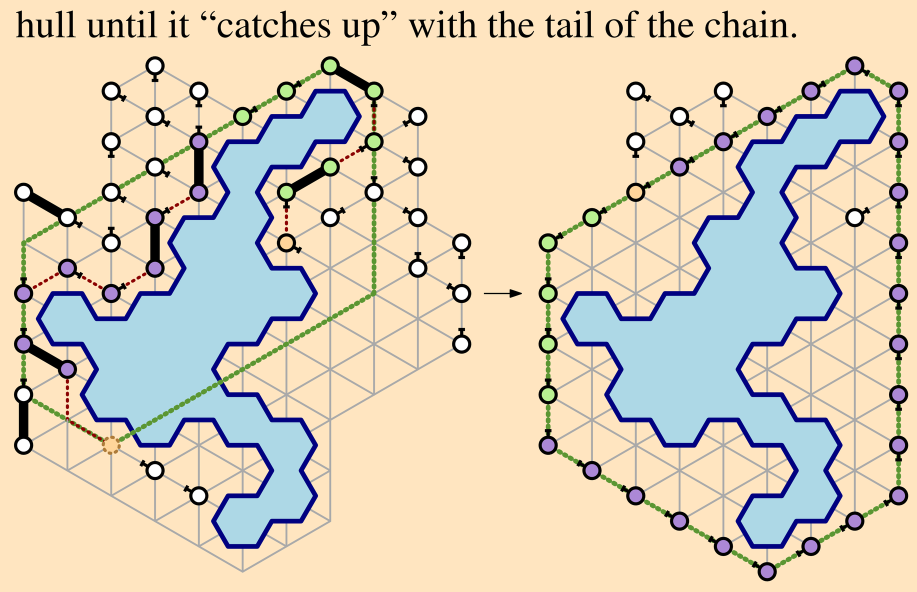
Leader election: A leader is determined and a spanning tree of the followers towards the leader is computed.

Counter: The counters can be maintained in the first $O(\log n)$ followers of the leader in a distributed fashion by passing tokens corresponding to bits in the binary representation of the counters.

Theorem 1. Given any fair asynchronous activation sequence of the robots, and any nonnegative sequence of m increment/decrement operations, the distributed binary counter can process all operations in $O(m)$ asynchronous rounds.

Phase 1: The leader walks along the perimeter of the object, maintaining the distances to the convex hull tangents.

Phase 2: The leader walks along the computed convex hull until it "catches up" with the tail of the chain.



Theorem 2. A set of robots can form a convex hull of an object in $O(B + H \log H)$ asynchronous rounds in the worst case, where B is the length of the perimeter of the object, and H is the length of its convex hull.

[1] J. Daymude, R. Gmyr, K. Hinnenthal, I. Kostitsyna, C. Scheideler, A. Richa. Convex Hull Formation for Programmable Matter. ArXiv:1805.06149, 2018

[2] Z. Derakhshandeh, S. Dolev, R. Gmyr, A. Richa, C. Scheideler, T. Strothmann. Brief announcement: Amoebot—a new model for programmable matter. In *26th ACM Symposium on Parallelism in Algorithms and Architectures (SPAA)*, 2014

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